A GENERALIZED DIGRAPH SIMULATOR

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ABSTRACT

Directed Graphs (Digraphs) can be used as powerful tools in the simulation of many social, political and economic models. Because the digraph can be thought of as a matrix and a transformation as simply a matrix multiplication, *APL* lends itself quite nicely to the implementation of a digraph simulation system. A collection of *APL* programs have been written (called GENSIM) that allow for the easy interactive definition correction, revision, running and outputting of digraph models. This paper gives a brief introduction to digraphs in apl terminology, a discussion of GENSIM and a sample terminal session showing the use of GENSIM.

DIRECTED GRAPHS AND WEIGHTED DIRECTED GRAPHS

At least in the beginning a directed graph (digraph) is best conceptualized visually. Consider the following digraph:

```
 -------→|‾A‾|--------
 |        ‾|‾        |
_|_        |        _↓_
|_C_|←--------      |_B_|
‾|‾                  ‾↑‾
 ---------------------
```

FIGURE 1.

The boxes represent variables (or nodes) while the arrows are called arcs. The interpretation of an arc is that a change in the value of a variable (referred to as a pulse) at the tail of an arc will have an effect upon the variable at its head. For example, in figure 1 above *A* has an effect upon variables *B* and *C* while variable *B* effects no other variables.

This sort of digraph really says very little about the interaction of the variables. It does not even say if a change in a variable will produce an effect in the same direction (a positive effect) or a change in an opposite direction (a negative effect). Often it is appropriate to assign a sign (+ or -) to an arc in order to clarify the nature of the effect that the arc represents. Even more complicated interactions can be described by assigning a number to each arc. This number is multiplied by the pulse at the tail of the arc to determine the pulse to be generated at the head. This is called a weighted digraph.

Consider the following example:

```
 --(¯.9)-→|‾A‾|--(¯.7)--
 |         ‾|‾         |
_|_         |         _↓_
|_C_|←-(.5)---        |_B_|
‾|‾                    ‾↑‾
 ------------(1.2)------
```

FIGURE 2.

Let us assume that all of the variables begin with a value of 10. If no variables change (are pulsed) then the graph says that all of the variables remain at a value of 10 for all future times. However if variable *A* receives a pulse of 5 it will go up in value to 15 however that pulse of 5 will produce a corresponding pulse of 2.5 in *C* and ¯3.5 in *B*. These pulses will propagate. The new pulse of 2.5 on *C* will increase the value of *C* to 12.5 but it will also produce a pulse of ¯2.25 upon *A* and a pulse of 3 upon *B*. The pulse of ¯3.5 on *B* will be lost since the graph shows no arcs that have tails at *B*.

The following table lists the values and pulses as they are generated by the model after the first 5 applications of the graph.

	PULSES			*VALUES*		
T	*A*	*B*	*C*	*A*	*B*	*C*
0	5	0	0	15	10	10
1	0	¯3.5	2.5	15	6.5	12.5
2	¯2.25	3	0	12.75	9.5	12.5
3	0	1.575	¯1.125	12.75	11.08	11.38
4	1.013	¯1.35	0	13.76	9.725	11.38
5	0	¯0.7088	0.5063	13.76	9.016	11.88

Notice that the value for a variable at a given time is equal to the value for the variable for the previous time plus the pulse on the variable for the current time.

The relationships expressed in a graph can also be expressed numerically in something called a weighted cross-impact matrix. The following is the cross-impact matrix for the graph in figure 2.

T	*A*	*B*	*C*
A	0	0	¯0.9
B	¯0.7	0	1.2
C	0.5	0	0

T[*I*;*J*] is the weight that applies to the effect of the *J*-th variable upon the *I*-th. Hence the next set of values for the variables (expressed as a vector of their values) is derived from the last set of pulses (also expressed as a vector) and cross-impact matrix (*T*) by the following *APL* expression:

```
NEXTVALUES←LASTVALUES+T+.×LASTPULSE
```

THE SIMULATOR (GENSIM)

The calculation of pulses for a grpah is a very simple process. However it may be very tedious to do by hand. GENSIM is a collection of *APL* programs that were written to allow the user to interactively build and modify digraph models. GENSIM was designed to meet the following performance criterion:

(1) There should be no limit to the number of variables that the model can handle (except for machine storage limits).

(2) Definition of the model must be simple for the person with no knowledge of either *APL* or the mathematics behind the model.

(3) Pulses and values can be changed mid-stream in order to reflect policy decisions concerning the model.

(4) The model can be backed up any number of time units in order to correct for bad decisions.

(5) Output must be available for in both tabular and graphic form.

In order to use GENSIM, the user must specify the following:

A Title
A time unit (the time that it takes for one application of the graph)
The number of variables in the model
A name for each of the variables (and a legend)
The cross-impact matrix (though the user need not know that he/she is building a matrix)
An initial pulse

Once this information is provided to the model, the user may run the model with the command:

```
FOREWARD N
```

which applies the graph to the values and pulses for *N* time units. Functions called *PRINT* and *PLOT* are available and they take arguments which describe the variables and time intervals for which the print or plot are to be given. Other commands are available for backing up the model, change pulses or values, modify the cross-impact matrix, display the status of the model, trace calculations, perform statistics, and list the arcs and their weights.

CONCLUSIONS

While the weighted digraph is not a particularly sophisticated modeling tool, often it is capable of expressing as much as is known about a system of variables. This is most often the case when the variables are not easily quantifiable. The building and use of a model is at its best a highly interactive process which incorperates many changes of assumptions and back-tracking. GENSIM is particularly good to the user and it is designed to be (in fact it has to be) used interactively.

Most of GENSIM's sophistication is aimed at input and output. There are still many things that yet could be done to increase the mathematical sophistication of the model. Some of them are:

(1) Time lags. Many effects are not felt for extremely long periods of time. Short time lags can presently be incorporated by using dummy variables but this is not an acceptable general solution.

(2) Pulses determined by non-linear combinations of previous pulses and values.

(3) Make use of available file systems. The current version was written for an *APL*\360 system without files.

Further Reading:

The GENSIM package was used by Dr. Fred Roberts of Rutgers University in the preparation of the report Weighted Digraph Models for Energy Use and Air Pollution in Transformation Systems, R-1578-NSF, December 1974, Rand Corporation. This report gives an excellent introduction to weighted digraphs. In addition a user's manual entitled GENSIM--A Generalized Digraph Simulator is available from the author.

APPENDIX I -- GENSIM USER FUNCTIONS

Function	Description
BACKWARD n	Same as *FOREWARD* -n except that both negative and positive arguments will back up the model n units.
ENTERNAMES	Defines the legend.
FOREWARD n	Applies the cross impact matrix enough times to advance the model n time units.
INITIALIZE	Resets time to 0 and sets pulse and values to those last set by the functions *SETPULSES*, *SETVALUES*, or *START*.
LEGEND	Displays the legend.
PLOT selection	Plots the values or pulses for selected variables for selected times. Selection is the argument to *PLOT* and it can be generated by using many combinations of other functions. Refer to the User's Manual.
PLOTALLP	Plots all pulses for all times.
PLOTALLV	Plots all values for all times.
PRINT selection	Prints the values or pulses for selected variables for selected times. Refer to the User's Manual.
PRINTALLP	Prints all pulses for all times.
PRINTALLV	Prints all values for all times.
RULES	Displays the non-zero elements of the cross-impact matrix.
SETPULSES	Allows the user to set the pulses. Useful for changing the pulses on some variables during the middle of the simulation.
SETTITLE	Allows the user to change the title.
SETVALUES	Allows the user to set current values. Also useful during the middle of a simulation.
SPARSE	Used to modify or check individual elements in the cross-impact matrix.
STATISTICS	Prints some descriptive statistics about selected parts of accumulated tables of values or pulses.

START Sets up initial model.

STATUS Displays information concerning the current status of the model.

TRACE Sets traces on parts of the intermediate calculations performed by the model.

*UMOD*1 Initially null function which can be modified by the user. *UMOD*1 is executed every time *FOREWARD* does a transformation.

*UMOD*2 Like *UMOD*1 except that it is executed every time a time interval is completed.

APPENDIX II -- SAMPLE TERMINAL SESSION

Note: Underlined text is typed by the user.

```
      )LOAD 1234 GENSIM
SAVED  14.41.16 03/12/76
      START
DO YOU WANT INSTRUCTIONS? (YES/NO)
NO
GIVE ME A ONE LINE TITLE FOR ALL THIS
SAMPLE DIGRAPH, SEE FIGURE 2 OF PAPER
WHAT IS YOUR UNIT OF TIME? (SINGULAR)
WEEK
HOW MANY TRANSFORMATIONS PER WEEK? (INTEGER ≥1)
⎕:
      1
HOW MANY VARIABLES WILL YOUR MODEL CONSIDER?
⎕:
      3
DEFINE A LEGEND, USE UP TO 50 CHARACTERS.
ENTRY FOR VARIABLE (1):
A↔APPLES
ENTRY FOR VARIABLE (2):
B↔BALOONS
ENTRY FOR VARIABLE (3):
C↔CARS
TRANSFORMATION RULES FOR PULSES
PICK: LONG, SHORT , OR SPARSE METHOD OF ENTERING TRANSFORMATION RULES
(TYPE: LONG/SHORT/SPARSE--OR NONE)
⎕:
      SPARSE
TYPE: → TO STOP
EFFECT OF: A
UPON: B
WAS:  0 NOW:
⎕:
      ¯.7
EFFECT OF: A
UPON: C
WAS:  0 NOW:
⎕:
      .5
EFFECT OF: C
UPON: B
WAS:  0 NOW:
⎕:
      1.2
EFFECT OF: C
UPON: A
WAS:  0 NOW:
⎕:
      ¯.9
EFFECT OF: →

ENTER VALUES: A,B,C
⎕:
      10 10 10
ENTER PULSES A,B,C
⎕:
      5 0 0
START IS FINISHED, DON'T FORGET TO )SAVE THIS WORKSPACE
```

FOREWARD 20
PRINT ALLVARIABLES INVTABLE FOR ALLTIMES EVERY 5

SAMPLE DIGRAPH, SEE FIGURE 2 *OF PAPER*

	VALUES		
WE	*A*	*B*	*C*
0	15	10	10
5	13.76	9.016	11.88
10	13.42	9.649	11.76
15	13.44	9.667	11.72
20	13.45	9.655	11.72

LEGEND

WE↔*TIME IN WEEKS*
A↔*APPLES*
B↔*BALOONS*
C↔*CARS*

PRINT 'AC' INPTABLE FOR 0 *THRUTIME* 5

SAMPLE DIGRAPH, SEE FIGURE 2 *OF PAPER*

	PULSE	
WE	*A*	*C*
0	5	0
1	0	2.5
2	¯2.25	0
3	0	¯1.125
4	1.013	0
5	0	0.5063

LEGEND

WE↔*TIME IN WEEKS*
A↔*APPLES*
C↔*CARS*

STATUS
SAMPLE DIGRAPH, SEE FIGURE 2 *OF PAPER*
TIME INTERVAL: *WEEKS*
CURRENT TIME: 20
NO. OF VARS: 3
TRANSFORMATIONS PER WEEK: 1
NO. T≠0: 4
TRACE ON SUMS OF POWERS: 0
TRACE ON POWERS OF T: 0
THESE TWO TRACES ARE USEABLE
TRACE ON VALUES: 0
TRACE ON PULSES: 0
SIG. FIGS. IN DISPLAY: 4
PAPER WIDTH:80
PLOT DEPTH: 25

```
      PLOTALLV
                 SAMPLE DIGRAPH, SEE FIGURE 2 OF PAPER

     15.0 A A
          |
          |        A A
        . |            A A A A A A A A A A A A A A A
          |    A A
     12.5 + C C
          |         C C       C C
 V        |      C C     C C     C C C C C C C C C C
 A        |      B
 L        |
 U   10.0 C              B
.E        |    B   B   B   B B B B B B B B B B B B B
 S        |          B
          |
          |
      7.5 +
          |
          | B
          |
          |
      5.0 +----'----'----'----'----'----'----'----'
          0         5         10        15        20
                      TIME (IN WEEKS)

LEGEND

A↔APPLES
B↔BALLOONS
C↔CARS

      FOREWARD ¯10
THE TIME IS WEEK NUMBER 20.
I NOW GO BACKWARD
ALL POWER AND SUMS OF POWERS OF T LOST
NOW WEEK 10

      PRINT ALLVARIABLES INPTABLE FOR 10 11 12
TIMES OUR OF BOUNDS (LESS THAN 0 OR GREATER THAN 10) IGNORED

    SAMPLE DIGRAPH, SEE FIGURE 2 OF PAPER
                  PULSE
     WE             A               B               C
     10          ¯0.09226         0.123             0
LEGEND
WE↔TIME IN WEEKS
A↔APPLES
B↔BALLOONS
C↔CARS

      SETPULSES
ENTER PULSES: A,B,C
⎕:
      ¯.09226 .123
ENTER PULSE: C
⎕:
      ¯6

      FOREWARD 10
```

PLOT 'BC' INVTABLE FOR ALLTIMES

```
               SAMPLE DIGRAPH, SEE FIGURE 2 OF PAPER

      15 +
         |                        C C     C C C C C
         | C C                        C C
         |         C C C C.C    C
         |     C C
      10 +   B   B   B B B B B
         |         B
 V       |
 A       | B
 L       |                   C
 U     5 +
 E       |                            B
 S       |                      B
         |                          B   B   B B B B
         |                                B
       0 +
         |                        B
         |
         |
         |
      ¯5 +----+----+----+----+----+----+----+----+
         0         5        10        15        20

                     TIME (IN WEEKS)
```

LEGEND

B↔*BALLOONS*
C↔*CARS*

RULES

A→*B*: ¯0.7
A→*C*: 0.5
A←*C*: ¯0.9

B←*A*: ¯0.7
B←*C*: 1.2

C→*A*: ¯0.9
C←*A*: 0.5
C→*B*: 1.2